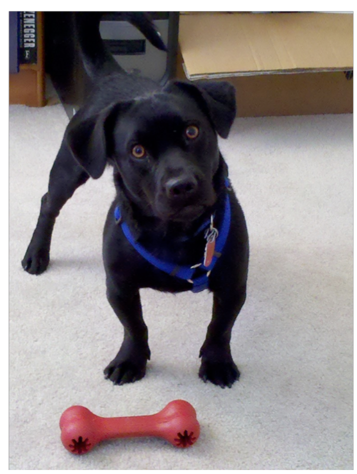
CSCI211: Intro Objectives

- Introduction to Algorithms, Analysis
- Course summary
- Reviewing proof techniques

My Bio

- From Dallastown, PA
- B.S., Gettysburg College
- M.S., Duke University
- Ph.D., University of Delaware
- For fun: pop culture, gardening, volunteer at Rockbridge Animal Alliance



What This Course Is About

From
30 Rock

Now, everything comes down to expert knowledge of **algorithms** and **data structures**.
If you don't speak fluent **O-notation**, you may have trouble getting your next job at the technology companies in the forefront.
-- Larry Freeman

For me, great algorithms are the poetry of computation. Just like verse, they can be terse, allusive, dense, and even mysterious. But once unlocked, they cast a **brilliant new light** on some aspect of computing.
-- *Francis Sullivan*

Motivation Google

- From a Google interview preparation email

Get your algorithms straight (they may comprise up to a **third** of your interview).
Visit: http://en.wikipedia.org/wiki/List_of_algorithm_general_topics
and examine this list of algorithms:
http://en.wikipedia.org/wiki/List_of_algorithms
and data structures: http://en.wikipedia.org/wiki/List_of_data_structures
Write out all the algorithms yourself from start to finish and make sure they're working.

What is an Algorithm?

- Precise procedure to solve a problem
- Completes in a finite number of steps

Questions to Consider

- What are our goals when designing algorithms?
- How do we know when we've met our goals?

- Goals: Correctness, Efficiency
- Use analysis to show/prove

Course Goals

- Learn how to formulate precise problem descriptions
- Learn specific algorithm design techniques and how to apply them
- Learn how to analyze algorithms for efficiency and for correctness
- Learn when no exact, efficient solution is possible

Course Content

- Algorithm analysis
  - Formal – proofs; Asymptotic bounds
- Advanced data structures
  - e.g., heaps, graphs
- Greedy Algorithms
- Divide and Conquer
- Dynamic Programming
- Network Flow
- Computational Intractability

Course Notes

- Textbook: *Algorithm Design*
- Participation is encouraged
  - Individual, row, class
- Assignments:
  - Reading text, writing brief summaries
    - Readings through Friday due following Tuesday
  - Problem Sets
    - Solutions to problems
    - Analysis of solutions
    - Programming (little)

Given on Friday, due following Friday

Course Grading

- 40% Individual written and programming homework assignments
- 30% Two midterm exams
- 20% Final
- 5% Text book reading summaries, weekly
  - In a journal on wiki
- 5% Participation and attendance

Text Book Summaries in Journal

- Important for you to wrestle with material more than just during the class period
  - More than superficial understanding
    - Understand problem, motivation, key insights, proof, analysis, …
  - Make connections
  - Not all details can be covered in class
- Not just reading → *active* summaries
- Help you prepare for the week's problem set

Journal Content

- Brief summary of chapter/section
  - ~1 paragraph of about 5-10 sentences/section
  - feel free to write more if that will help you
- Include motivations for the given problem, as appropriate
- For algorithms, brief sketch of algorithm, intuition, and implementation
  - Include runtime
- Questions you have about motivation/solution/proofs/analysis
- Discuss anything that makes more sense after reading it again, after it was presented in class (or vice versa)
- Anything that you want to remember, anything that will help you
- Say something about how readable/interesting the section was on scale of 1 to 10

Journal Grading

Grade	Meaning
+	Especially well-done, insightful questions
	Typical grade
-	Unsatisfactory write up; specific feedback about how to improve
0	No submission

How to Succeed in This Course

- Come to every class prepared (bring questions!)
- Actively participate in class by asking and answering questions
- Actively read the textbook, making notes about the problems and solutions in your wiki.
- Do all the assignments--start them when they are assigned--and turn them in on time
  - Refer to your wiki, the lecture slides, and your notes when working on your assignments.
- If you start to get behind, see me in office hours right away

http://cs.wlu.edu/~sprenkle/cs211

ALGORITHMS

Computational Problem Solving 101

- Computational Problem
  - A problem that can be solved by logic

- To solve the problem:
  1. Create a *model* of the problem
  2. Design an *algorithm* for solving the problem using the model
  3. Write a *program* that implements the algorithm

Computational Problem Solving 101

- Algorithm: a well-defined recipe for solving a problem
  - Has a finite number of steps
  - Completes in a finite amount of time

- Program
  - An algorithm written in a programming language
  - Important to consider implementation's effect on runtime

PROOFS

Why Proofs?

- What are insufficient alternatives?
- How can we prove something isn't true?

Why Proofs?

- What are insufficient alternatives?
  - Examples
    - Considered all possible?
  - Empirical/statistical evidence
    - Ex: “Lying” with statistics
- How can we prove something isn’t true?
  - One counterexample

Need irrefutable proof that something is true—for ***all*** possibilities

Soap Op

- “It’s the

Common proof techniques

Proof by intimidation Trivial!

Proof by cumbersome notation The theorem follows immediately from the fact that $\left|\bigoplus_{k\in S}(\mathfrak{K}^{\mathbb{F}^{\alpha}(i)})_{i\in\mathcal{U}_k}\right| \preccurlyeq \aleph_1$ when $[\mathfrak{H}]_{\mathcal{W}} \cap \mathbb{F}^{\alpha}(\mathbb{N}) \neq \emptyset$.

Proof by inaccessible literature The theorem is an easy corollary of a result proven in a hand-written note handed out during a lecture by the Yugoslavian Mathematical Society in 1973.

Proof by ghost reference The proof my be found on page 478 in a textbook which turns out to have 396 pages.

Circular argument Proposition 5.18 in [BL] is an easy corollary of Theorem 7.18 in [C], which is again based on Corollary 2.14 in [K]. This, on the other hand, is derived with reference to Proposition 5.18 in [BL].

Proof by authority My good colleague Andrew said he thought he might have come up with a proof of this a few years ago...

Internet reference For those interested, the result is shown on the web page of this book. Which unfortunately doesn’t exist any more.

Proof by avoidance *Chapter 3:* The proof of this is delayed until Chapter 7 when we have developed the theory even further. *Chapter 7:* To make things easy, we only prove it for the case $z = 0$, but the general case in handled in Appendix C. *Appendix C:* The formal proof is beyond the scope of this book, but of course, our intuition knows this to be true.

facebook.com/Mathematicx

Common Types of Proofs?

Common Types of Proofs

- Direct proofs
  - Series of true statements, each implies the next
- Proof by contradiction
- Proof by induction

Proof By Contradiction

What are the steps to a proof by contradiction?

Proof By Contradiction

1. Assume the proposition (P) we want to prove is false
2. Reason to a contradiction
3. Conclude that P must therefore be true

Prove: There are Infinitely Many Primes

Prove: There are Infinitely Many Primes

- What is a prime number?
- What is not-a-prime number?

- What is our first step (proof by contradiction)?
- What do we want to show?

Prove: There are Infinitely Many Primes

- Assume there are a finite number of prime numbers
  - List them: p_1, p_2 ..., p_n
- Consider the number $q = p_1p_2... p_n + 1$

What are the possibilities for q?

q is either composite or prime

Prove: There are Infinitely Many Primes

- Assume there are a finite number of prime numbers
  - List them: p_1, p_2 ..., p_n
- Consider the number $q = p_1p_2... p_n + 1$
- Case: q is composite
  - If we divide q by any of the primes, we get a remainder of 1 → q is not composite

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- Case: q is composite
  - If we divide q by any of the primes, we get a remainder of 1 → q is not composite
- Therefore, q is prime, but q is larger than any of the finitely enumerated prime numbers listed → *Contradiction*

Proof thanks to Euclid

Proof By Induction

What are the steps to a proof by induction?

Proof By Induction

1. What you want to prove
2. Base case
   - Typical: Show statement holds for $n = 0$ or $n = 1$
3. ***Induction hypothesis***
4. Induction step: show that adding one to n also holds true
   - Relies on earlier assumptions

When/why is induction useful?

Show true for all (infinite) possibilities
Show works for "one more"

Proof By Induction

1. State your P(n).
   - P(n) is a property as a function of *n*
     - State for which *n* you will prove your P(n) to be true
2. State your base case.
   - State for which n your base case is true, and prove it
     - Use the smallest *n* for which your statement is true
3. State your induction hypothesis
   - Without an induction hypothesis, the proof falls apart.
   - Usually it is just restating your P(n), with no restriction on *n* (an arbitrary n)
4. Inductive Step.
   - Consider P(n + 1).
     - Try to prove a larger case of the problem than you assumed in your induction hypothesis.
   - Keep in mind: What are you trying to prove?
   - Use your induction hypothesis, and clearly state where it is used.
     If you haven't used your induction hypothesis, then you are not doing a proof by induction.
5. Conclusion.
   - Optionally, restate the problem.

Example of Induction Proof

Prove:
2+4+6+8+… + 2n = n*(n+1)

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2+4+6+8+… + 2n = n*(n+1)

For what values of *n* do we want to prove this is true?

A: where *n* is a natural number

Example of Induction Proof

Prove: 2+4+6+8+… + 2n = n*(n+1)

(where *n* is a natural number)

- **Base case**: *n* = 1 →
  - 2*1 = 1*(1+1)

Example of Induction Proof

Prove: 2+4+6+8+… + 2n = n*(n+1)

(where *n* is a natural number)

- Base case: *n* = 1 →
  - 2*1 = 1*(1+1)
- **Induction Hypothesis:**
  - Assume statement is true for some arbitrary k > 1

Example of Induction Proof

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(where *n* is a natural number)

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- Base case: *n* = 1 →
  - 2*1 = 1*(1+1)
- Induction Hypothesis:
  - Assume statement is true for some arbitrary k > 1
- Prove holds for *k*+1, i.e., show that 2+4+6+8+… + 2k + 2(k+1) = (k+1)*((k+1)+1)

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- Prove true for *k*+1, i.e., show that
  2+4+6+8+… + 2k + 2(k+1) = (k+1)*((k+1)+1)
  - 2+4+6+8+… + 2k + 2(k+1)

  = k*(k+1) + 2(k+1)

  = k^2 + k + 2k + 1

  = k^2 + 3k + 1

  = (k+1)*(k+2)

  = (k+1)*((k+1)+1)

Approach shown: transform LHS to RHS

I want to see these steps in your proofs!

Prove: 2+4+6+8+… + 2n = n*(n+1)

- Base case: *n* = 1 → 2*1 = 1*(1+1)
- Assume statement is true for arbitrary *n*=*k*>1
- Prove true for *k*+1, i.e., show that
  2+4+6+8+… + 2k + 2(k+1) = (k+1)*((k+1)+1)
  - 2+4+6+8+… + 2k + 2(k+1)

  = k*(k+1) + 2(k+1)

  = (k+1)*(k+2), factor out the (k+1)

  = (k+1)*((k+1)+1)

Alternative solution

Proof Summary

- Need to ***prove*** conjectures
- Common types of proofs
  - Direct proofs
  - Contradiction
  - Induction
- Common error: not checking/proving assumptions
  - "Jumps" in logic

Proof: All Horses Are The Same Color

- **Base case**: If there is only *one* horse, there is only one color.
- **Induction step**: Assume as induction hypothesis that within any set of n horses, there is only one color.
  - Look at any set of $n + 1$ horses
  - Label the horses: 1, 2, 3, ..., n, $n + 1$
  - Consider the sets {1, 2, 3, ..., n} and {2, 3, 4, ..., $n + 1$}
  - Each is a set of only n horses, therefore within each there is only one color
  - Since the two sets overlap, there must be only one color among all $n + 1$ horses

Where is the error in the proof?

Looking Ahead

- Check out course wiki page
  - Test username/password after email received
  - Decide which style of journal you want: wiki or blog
- Read first two pages of book's preface
  - Summarize on Wiki by next Tuesday @ midnight